Int. Journal of Math. Analysis, Vol. 7, 2013, no. 8, 349 - 359

Contra Pre-$\mathcal{I}$-Continuous Functions

T. Noiri

2949-1 Shiokita-cho, Hinagu, Yatsushiro-shi
Kumamoto-ken, 869-5142 Japan
t.noiri@nifty.com

S. Jafari

College of Vestsjaelland South, Herrestrade 11
4200 Slagelse, Denmark
jafaripersia@gmail.com

K. Viswanathan and J. Jayasudha

Post-Graduate and Research Department of Mathematics, N G M College
Pollachi - 642 001, Tamil Nadu, India
visungm@yahoo.com

Abstract

In this paper, we apply the notion of pre-$\mathcal{I}$-open sets to present and study a new class of functions called contra pre-$\mathcal{I}$-continuous functions in ideal topological spaces. Relationships between this new class and other classes of functions are investigated and some characterisations of this new class of functions are studied. Also we introduce the notions of contra strong β-$\mathcal{I}$-continuous functions and contra δ-$\mathcal{I}$-continuous functions to obtain decompositions of contra α-$\mathcal{I}$-continuity and contra semi-$\mathcal{I}$-continuity.

Mathematics Subject Classification: Primary 54A05; Secondary 54D15, 54D30.

Keywords: pre-$\mathcal{I}$-open set, contra pre-$\mathcal{I}$-continuity, contra semi-$\mathcal{I}$-continuity, contra α-$\mathcal{I}$-continuity, contra β-$\mathcal{I}$-continuity

1 Introduction and Preliminaries

An ideal $\mathcal{I}$ on a topological space (X, τ) is a non-empty collection of subsets of X satisfying the following properties:(1) $A \in \mathcal{I}$ and $B \subseteq A$ imply $B \in \mathcal{I}$

(heredity); (2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$ (finite additivity). A topological space (X, τ) with an ideal $\mathcal{I}$ on X is called an *ideal topological space* and is denoted by $(X, \tau, \mathcal{I})$. For a subset $A \subseteq X$, $A^*(\mathcal{I}) = \{x \in X : U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$, is called the *local function* [13] of A with respect to $\mathcal{I}$ and τ. We simply write A^* in case there is no chance for confusion. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(\mathcal{I})$ called the $*$-topology finer than τ is defined by $cl^*(A) = A \cup A^*$ [26]. Let (X, τ) denote a topological space on which no separation axioms are assumed unless explicitly stated. In a topological space (X, τ), the closure and the interior of any subset A of X will be denoted by $cl(A)$ and $int(A)$, respectively. The kernel [14] of A, denoted by $\ker(A)$, is the intersection of all open supersets of A. A subset A of a topological space (X, τ) is said to be pre-open [15]) if $A \subseteq int(cl(A)))$.

Lemma 1.1.. [11] The following properties hold for subsets A, B of a topological space (X, τ) :

1. $x \in ker(A)$ if and only if $A \cap F \neq \phi$ for any closed set F of X containing x;
2. $A \subset ker(A)$ and $A = ker(A)$ if A is open in X;
3. If $A \subset B$ then $ker(A) \subset ker(B)$.

Definition 1.2.. A topological space (X, τ) is said to be

1. extremally disconnected [23] if the closure of every open set of X is open in X,
2. submaximal [19] if every dense set of X is open in X, equivalently if every preopen set is open.

Definition 1.3.. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

1. almost pre-continuous [17] if for each $x \in X$ and each open neighbourhood V of $f(x)$, there exists $U \in PO(X)$ containing x such that $f(U) \subset int(cl(V))$.
2. contra-continuous [6] (briefly c-c) if $f^{-1}(V)$ is a closed set in X for every open set V of Y.
3. contra-pre-continuous [12] (briefly c-pre-c) if $f^{-1}(V)$ is a preclosed set in X for every open set V of Y.
4. contra-β-continuous [5] (briefly c-β-c) if $f^{-1}(V)$ is a β-closed set in X for every open set V of Y.

Definition 1.4.. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be

1. pre-$\mathcal{I}$-open [7] if $A \subseteq int(cl^*(A))$.
2. semi-$\mathcal{I}$-open [9] if $A \subseteq cl^*(int(A))$.
3. α-$\mathcal{I}$-open [9] if $A \subseteq int(cl^*(int(A)))$.
4. β-$\mathcal{I}$-open [9] if $A \subseteq cl(int(cl^*(A)))$.
5. strong β-$\mathcal{I}$-open [10] if $A \subseteq cl^*(int(cl^*(A)))$.
6. δ-$\mathcal{I}$-open [2] if $int(cl^*(A)) \subseteq cl^*(int(A))$.

Definition 1.5.. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be pre-$\mathcal{I}$-continuous [7] (resp. semi-$\mathcal{I}$-continuous [9], α-$\mathcal{I}$-continuous [9], β-$\mathcal{I}$-continuous[9]) if for each open set V of (Y, σ), $f^{-1}(V)$ is a pre-$\mathcal{I}$-open (resp. semi-$\mathcal{I}$-open, α-$\mathcal{I}$-open, β-$\mathcal{I}$-open) set in $(X, \tau, \mathcal{I})$.

Definition 1.6.. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be

1. contra semi-$\mathcal{I}$-continuous [16] (briefly c-semi-$\mathcal{I}$-c) if $f^{-1}(V)$ is semi-$\mathcal{I}$-open in $(X, \tau, \mathcal{I})$ for every closed set V of (Y, σ).
2. contra α-$\mathcal{I}$-continuous [25] (briefly c-α-$\mathcal{I}$-c) if $f^{-1}(V)$ is α-$\mathcal{I}$-open in $(X, \tau, \mathcal{I})$ for every closed set V of (Y, σ).
3. contra β-$\mathcal{I}$-continuous [4] (briefly c-β-$\mathcal{I}$-c) if $f^{-1}(V)$ is β-$\mathcal{I}$-open in $(X, \tau, \mathcal{I})$ for every closed set V of (Y, σ).

2 Contra pre-$\mathcal{I}$-continuous functions

Definition 2.1.. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is called contra pre-$\mathcal{I}$-continuous (briefly c-pre-$\mathcal{I}$-c) (resp. contra strong β-$\mathcal{I}$-continuous (briefly c-strong β-$\mathcal{I}$-c), contra δ-$\mathcal{I}$-continuous) if $f^{-1}(V)$ is pre-$\mathcal{I}$-open (resp. strong β-$\mathcal{I}$-open, δ-$\mathcal{I}$-open) in $(X, \tau, \mathcal{I})$ for every closed set V of (Y, σ).

Proposition 2.2.. *The following statements hold for a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$:*

1. *Every contra-continuous function is contra α-$\mathcal{I}$-continuous* [25].
2. Every contra α-$\mathcal{I}$-continuous function is contra pre-$\mathcal{I}$-continuous.
3. Every contra pre-$\mathcal{I}$-continuous function is contra pre-continuous.

4. Every contra pre-$\mathcal{I}$-continuous function is contra strong β-$\mathcal{I}$-continuous.

5. Every contra strong β-$\mathcal{I}$-continuous function is contra β-$\mathcal{I}$-continuous.

Proof. This follows from Remark 2.1 of [9].

Remark 2.3.. *The notions of pre-$\mathcal{I}$-continuity and contra pre-$\mathcal{I}$-continuity are independent.*

Remark 2.4.. *We have the following implications:*

$$\begin{array}{ccccccc}
\text{c-c} & & & & & & \\
\Downarrow & & & & & & \\
\text{c-}\alpha\text{-}\mathcal{I}\text{-c} & \Rightarrow & \text{c-pre-}\mathcal{I}\text{-c} & & \Longrightarrow & & \text{c-pre-c} \\
\Downarrow & & \Downarrow & & & & \Downarrow \\
\text{c-semi-}\mathcal{I}\text{-c} & \Rightarrow & \text{c-strong } \beta\text{-}\mathcal{I}\text{-c} & \Rightarrow & \text{c-}\beta\text{-}\mathcal{I}\text{-c} & \Rightarrow & \text{c-}\beta\text{-c}
\end{array}$$

Theorem 2.5.. *For a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following are equivalent:*

1. *f is contra pre-$\mathcal{I}$-continuous.*

2. *For every closed subset F of Y, $f^{-1}(F) \in PIO(X)$.*

3. *For each $x \in X$ and each closed set F of Y containing $f(x)$, there exists $U \in PIO(X)$ such that $f(U) \subset F$.*

4. *$f(p\mathcal{I}cl(A)) \subset ker(f(A))$ for every subset A of X.*

5. *$p\mathcal{I}cl(f^{-1}(B)) \subset f^{-1}(ker(B))$ for every subset B of Y.*

Proof. The implications (1)⇒(2) and (2)⇒(3) are obvious.
(3)⇒(2): Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in PIO(X)$ containing x such that $f(U_x) \subset F$. Therefore, we obtain $f^{-1}(F) = \cup\{U_x | x \in f^{-1}(F)\}$. Hence $f^{-1}(F) \in PIO(X)$.
(2)⇒(4): Let A be any subset of X. Suppose that $y \notin ker(f(A))$. Then by Lemma 1.1, there exists a closed set F of Y containing y such that $f(A) \cap F = \phi$. Thus, we have $A \cap f^{-1}(F) = \phi$ and $p\mathcal{I}cl(A) \cap f^{-1}(F) = \phi$. Therefore, we obtain $f(p\mathcal{I}cl(A)) \cap F = \phi$ and $y \notin f(p\mathcal{I}cl(A))$. This implies that $f(p\mathcal{I}cl(A)) \subset ker(f(A))$.
(4)⇒(5): Let B be any subset of Y. By (4) and Lemma 1.1, we have $f(p\mathcal{I}cl(f^{-1}(B))) \subset ker(f(f^{-1}(B))) \subset ker(B)$ and $p\mathcal{I}cl(f^{-1}(B)) \subset f^{-1}(ker(B))$.
(5)⇒(1): Let V be any open set of Y. Then by Lemma 1.1, we have $p\mathcal{I}cl(f^{-1}(V)) \subset f^{-1}(ker(V)) = f^{-1}(V)$ and $p\mathcal{I}cl(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is pre-$\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$.

Theorem 2.6.. *If a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra pre-$\mathcal{I}$-continuous and Y is regular, then f is pre-$\mathcal{I}$-continuous.*

Proof. Let x be an arbitrary point of X and V be an open set of Y containing $f(x)$. Since Y is regular, there exists an open set W in Y containing $f(x)$ such that $cl(W) \subset V$. Since f is contra pre-$\mathcal{I}$-continuous, by Theorem 2.5 there exists $U \in PIO(X)$ containing x such that $f(U) \subset cl(W)$. Then $f(U) \subset cl(W) \subset V$. Hence, f is pre-$\mathcal{I}$-continuous.

Corollary 2.7.. [12] If a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra pre-continuous and Y is regular, then f is pre-continuous.

Proof. The proof follows from Theorem 2.5, if $\mathcal{I} = \{\emptyset\}$.

Theorem 2.8.. *1. A function $f : (X, \tau, \{\emptyset\}) \to (Y, \sigma)$ is contra pre-$\mathcal{I}$-continuous if and only if it is contra pre-continuous.*
2. A function $f : (X, \tau, \mathcal{P}(X)) \to (Y, \sigma)$ is contra pre-$\mathcal{I}$-continuous if and only if it is contra pre-continuous.
3. A function $f : (X, \tau, \mathcal{N}) \to (Y, \sigma)$ is contra pre-$\mathcal{I}$-continuous if and only if it is contra continuous ($\mathcal{N}$ is the ideal of all nowhere dense sets).

Proof. The proof follows from Proposition 2.7 of [8].

Definition 2.9.. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be M-pre-$\mathcal{I}$-open if the image of each pre-$\mathcal{I}$-open set is pre-$\mathcal{I}$-open.

Theorem 2.10.. *If a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is M-pre-$\mathcal{I}$-open and contra pre-$\mathcal{I}$-continuous, then f is almost pre-continuous.*

Proof. Let x be any arbitrary point of X and V be an open neighbourhood of $f(x)$. Since f is contra pre-$\mathcal{I}$-continuous, by Theorem 2.5, there exists $U \in PIO(X)$ containing x such that $f(U) \subset cl(V)$. Since f is M-pre-$\mathcal{I}$-open, $f(U)$ is pre-$\mathcal{I}$-open in Y.Therefore $f(U) \subset int(cl^*(f(U)) \subset int(cl^*(cl(V))) \subset int(cl(V))$. This shows that f is almost pre-continuous.

Definition 2.11.. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be almost weakly-$\mathcal{I}$-continuous if $f^{-1}(V) \subset int(cl^*(f^{-1}(cl(V))))$ for every open set V of Y.

Theorem 2.12.. *If a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra pre-$\mathcal{I}$-continuous, then f is almost weakly-$\mathcal{I}$-continuous.*

Proof. Let V be any open set of Y. Since $cl(V)$ is closed in Y, $f^{-1}(cl(V))$ is pre-$\mathcal{I}$-open in X and we have $f^{-1}(V) \subset f^{-1}(cl(V)) \subset int(cl^*(f^{-1}(cl(V))))$. This shows that f is almost weakly-$\mathcal{I}$-continuous.

Theorem 2.13.. *The following are equivalent for a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$:*

1. *f is contra α-$\mathcal{I}$-continuous;*
2. *f is contra pre-$\mathcal{I}$-continuous and contra semi-$\mathcal{I}$-continuous.*

Proof. (1)⇒(2): Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be contra α-$\mathcal{I}$-continuous, then for every closed set V of Y, $f^{-1}(V)$ is an α-$\mathcal{I}$-open set in $(X, \tau, \mathcal{I})$. Now by Lemma 3.1 of [1], if follows that $f^{-1}(V)$ is pre-$\mathcal{I}$-open and semi-$\mathcal{I}$-open. Hence f is contra pre-$\mathcal{I}$-continuous and contra semi-$\mathcal{I}$-continuous.
(2)⇒(1): Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be contra pre-$\mathcal{I}$-continuous and contra semi-$\mathcal{I}$-continuous, then for every closed set V of Y, $f^{-1}(V)$ is both pre-$\mathcal{I}$-open and semi-$\mathcal{I}$-open. Again by Lemma 3.1 of [1], if follows that $f^{-1}(V)$ is α-$\mathcal{I}$-open in $(X, \tau, \mathcal{I})$. Thus f is contra α-$\mathcal{I}$-continuous.

Theorem 2.14.. *The following are equivalent for a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$:*

1. *f is contra semi-$\mathcal{I}$-continuous;*
2. *f is contra δ-$\mathcal{I}$-continuous and contra strong β-$\mathcal{I}$-continuous.*

Proof. The proof is similar to Theorem 2.13 and this follows by Proposition 2.1 of [2].

Theorem 2.15.. *The following are equivalent for a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$:*

1. *f is contra α-$\mathcal{I}$-continuous;*
2. *f is contra δ-$\mathcal{I}$-continuous and contra pre -$\mathcal{I}$-continuous.*

Proof. The proof is similar to Theorem 2.13 and this follows by Proposition 2.2 of [2].

Definition 2.16.. A space $(X, \tau, \mathcal{I})$ is said to be pre-$\mathcal{I}$-T_1 if for each pair of distinct points x and y in X, there exist pre-$\mathcal{I}$-open sets U and V containing x and y, respectively, such that $y \notin U$ and $x \notin V$.

Definition 2.17.. A space $(X, \tau, \mathcal{I})$ is said to be pre-$\mathcal{I}$-T_2 if for each pair of distinct points x and y in X, there exist pre-$\mathcal{I}$-open sets U and V containing x and y, respectively, such that $U \cap V = \phi$.

Theorem 2.18.. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Suppose that for each pair of distinct points x_1 and x_2 in X, there exists a function f of X into a Urysohn space Y such that $f(x_1) \neq f(x_2)$. Moreover, let f be contra pre -$\mathcal{I}$-continuous at x_1 and x_2. Then X is pre-$\mathcal{I}$-T_2.*

Corollary 2.19.. *If f is a contra pre-$\mathcal{I}$-continuous injection of an ideal topological space $(X, \tau, \mathcal{I})$ into a Urysohn space (Y, σ), then $(X, \tau, \mathcal{I})$ is a pre-$\mathcal{I}$-T_2 space.*

Proof. Suppose that $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is a contra pre-$\mathcal{I}$-continuous injection and Y is a Urysohn space. Then for each pair of distinct points x_1 and x_2 in X, $f(x_1) \neq f(x_2)$. Therefore, by Theorem 2.18, X is pre-$\mathcal{I}$-T_2.

Definition 2.20.. A space $(X, \tau, \mathcal{I})$ is said to be pre-$\mathcal{I}$-connected if X cannot be expressed as the disjoint union of two non-empty pre-$\mathcal{I}$-open sets.

Theorem 2.21.. *Let $(X, \tau, \mathcal{I})$ be pre-$\mathcal{I}$-connected and Y be a T_1-space. If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra pre-$\mathcal{I}$-continuous, then f is a constant function.*

Proof. Since Y is a T_1-space, $C = \{f^{-1}(\{y\}) | y \in Y\}$ is a disjoint partition of X by pre-$\mathcal{I}$-open sets. If $|C| \geq 2$, then X is the union of two non-empty pre-$\mathcal{I}$-open sets. Since X is pre-$\mathcal{I}$-connected, $|C| = 1$. Therefore f is constant.

Theorem 2.22.. *For a space $(X, \tau, \mathcal{I})$, the following are equivalent:*

1. *X is pre-$\mathcal{I}$-connected.*
2. *The only subsets of X which are both pre-$\mathcal{I}$-open and pre-$\mathcal{I}$-closed are the empty set ϕ and X.*
3. *Each contra pre-$\mathcal{I}$-continuous function of X into a discrete space Y with at least two points is a constant function.*

Theorem 2.23.. *If f is a contra pre -$\mathcal{I}$-continuous function from a pre-$\mathcal{I}$-connected space $(X, \tau, \mathcal{I})$ onto any space Y, then Y is not a discrete space.*

Theorem 2.24.. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is a contra pre -$\mathcal{I}$-continuous surjection and X is pre-$\mathcal{I}$-connected, then Y is connected.*

Definition 2.25.. The graph $G(f)$ of a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be contra pre -$\mathcal{I}$-closed in $X \times Y$ if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist a pre-$\mathcal{I}$-open set U in X containing x and a closed set V in Y containing y such that $(U \times V) \cap G(f) = \phi$.

Lemma 2.26.. *The graph $G(f)$ of a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra pre -$\mathcal{I}$-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist a pre-$\mathcal{I}$-open set U in X containing x and a closed set V in Y containing y such that $f(U) \cap V = \phi$.*

Theorem 2.27.. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is a contra pre -$\mathcal{I}$-continuous function and Y is a Urysohn space, then $G(f)$ is contra pre-$\mathcal{I}$-closed in $X \times Y$.*

Theorem 2.28.. *Let $(X, \tau, \mathcal{I})$ be any ideal topological space and let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a function and $g : X \to (X \times Y)$ be the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra pre -$\mathcal{I}$-continuous, then f is contra pre -$\mathcal{I}$-continuous.*

Definition 2.29.. A space (X, τ) is said to be weakly Hausdroff [21] if each element of X is an intersection of regular closed sets.

Theorem 2.30.. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is a contra pre -$\mathcal{I}$-continuous injection and Y is weakly Hausdroff, then X is pre -$\mathcal{I}$-T_1.*

Proof. Suppose that Y is weakly Hausdroff. For any distinct points x_1 and x_2 in X, there exist regular closed sets U and V in Y such that $f(x_1) \in U$, $f(x_2) \notin U$, $f(x_1) \notin V$ and $f(x_2) \in V$. Since f is contra pre -$\mathcal{I}$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are pre -$\mathcal{I}$-open subsets of X such that $x_1 \in f^{-1}(U)$, $x_1 \notin f^{-1}(V)$, $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$. This shows that X is pre -$\mathcal{I}$-T_1.

Theorem 2.31.. *Let the graph of a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be contra pre-$\mathcal{I}$-closed. If f is injective, then X is pre -$\mathcal{I}$-T_1.*

Theorem 2.32.. *Let (X, τ) be extremally disconnected and submaximal, then for any ideal $\mathcal{I}$ on X, the following are equivalent for a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$:*

1. *f is contra pre-$\mathcal{I}$-continuous,*
2. *f is contra pre-continuous,*
3. *f is contra semi-$\mathcal{I}$-continuous,*
4. *f is contra semi-continuous,*
5. *f is contra α-$\mathcal{I}$-continuous,*
6. *f is contra α-continuous,*
7. *f is contra-continuous.*

Proof. This follows from Corollary 4.42 of [3].
Recall, a topological space (X, τ) is locally irreducible if each point $x \in X$ has an irreducible neighbourhood.

Remark 2.33.. *It should be noticed that since every locally irreducible space is extremally disconnected, then if the space is locally irreducible and submaximal, then the conditions of Theorem 2.32 hold.*

Recall, a topological space (X, τ) is R_0 if every open set contains the closure of each of its singletons.

Remark 2.34.. *The Theorem 2.32 is true if the space is submaximal, R_0 and the intersection of any family of open subsets of (X, τ) is open.*

Definition 2.35.. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is said to be pre-$\mathcal{I}$-irresolute if $f^{-1}(V)$ is pre-$\mathcal{I}$-open in X for every pre-$\mathcal{J}$-open set V of Y.

Theorem 2.36.. *For the functions $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ and $g : (Y, \sigma, \mathcal{J}) \to (Z, \eta)$, the following hold:*

1. *$g \circ f$ is pre-$\mathcal{I}$-continuous if f is contra pre-$\mathcal{I}$-continuous and g is contra-continuous.*
2. *$g \circ f$ is contra pre-$\mathcal{I}$-continuous if f is contra pre-$\mathcal{I}$-continuous and g is continuous.*
3. *$g \circ f$ is pre-$\mathcal{I}$-continuous if f is pre-$\mathcal{I}$-irresolute and g is contra pre-$\mathcal{J}$-continuous.*

Remark 2.37.. *The composition of any contra pre-$\mathcal{I}$-continuous functions need not be contra pre-$\mathcal{I}$-continuous in general.*

References

[1] A. Acikgoz, T. Noiri and S. Yuksel, On α-$\mathcal{I}$-continuous and α-$\mathcal{I}$-open functions, Acta Math.Hungar., 105 (1-2)(2004), 27-37.

[2] A. Acikgoz, T. Noiri and S. Yuksel, On δ-$\mathcal{I}$-open sets and decomposition of α-$\mathcal{I}$-continuity, Acta Math.Hungar., 102 (4)(2004), 349-357.

[3] Metin Akdag, On b-$\mathcal{I}$-open sets and b-$\mathcal{I}$-continuous functions, Internat. J. Math. Math. Sci., (2007), Article ID 75721, 13 pages, doi:10.1155/2007/75721.

[4] J. Bhuvaneswari, A. Keskin, N. Rajesh, Contra-continuity via topological ideals, J. Adv. Res. Pure Math., 3(1)(2011), 40-51.

[5] M. Caldas, S. Jafari, Some properties of contra-β-continuous functions, Mem. Fac. Sci. Kochi Univ. (Math.), 22(2001), 19-28.

[6] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, Internet. J. Math. Math. Sci., 19(1996), 303-310.

[7] J. Dontchev, On pre-$\mathcal{I}$-open sets and a decomposition of $\mathcal{I}$-continuity, Banyan Math. J., 2(1996).

[8] J. Dontchev, Idealization of Ganster-Reilly decomposition theorems, Math. GN/9901017, 5 Jan. 1999(Internet).

[9] E. Hatir and T. Noiri, On decompositions of continuity via idealization, Acta Math. Hungar., 96(2002), 341-349.

[10] E. Hatir, A. Keskin and T. Noiri, On a new decomposition of continuity via idealization, JP Jour. Geometry and Topology, 1(2003), 53-64.

[11] S. Jafari and T. Noiri, Contra-super-continuous functions, Ann. Univ. Sci. Budapest, 42(1999), 27-34.

[12] S. Jafari and T. Noiri, Contra-precontinuous functions, Bull. Malaysian Math. Sci. Soc., (Second series) 25(2002), 115-128.

[13] K. Kuratowski, Topology, Vol. I, Academic press, New York, 1966.

[14] H. Maki, The special issue in commemoration of Prof. Jazusada IKEDA's retirement, 1. Oct. 1986, 139-146.

[15] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deep, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, 53(1982), 47-53.

[16] Jamal M. Mustafa, Contra semi-$\mathcal{I}$-continuous functions, Hacettepe J. Math. Stat., 39(2)(2010), 191-196.

[17] A. Nasef and T. Noiri, Some weak forms of almost continuity, Acta Math. Hungar., 74(1997), 211-219.

[18] T. Noiri, H. Maki and J. Umehara, Generalized preclosed functions, Mem. Fac. Sci. Kochi Univ. Ser. A Math., 19(1998), 13-20.

[19] I. L. Reilly and M. K. Vamanamurthy, On some questions concerning preopen sets, Kyungpook Math. J., 30(1990), 87-93.

[20] V. Renukadevi, Note on $\mathcal{I}$R-closed and $A_{\mathcal{I}R}$-sets, Acta Math. Hungar., 122(4)(2009), 329-338.

[21] T. Soundararajan, Weakly Hausdorff spaces and the cardinality of topological spaces, General Topology and its Relations to Modern Analysis and Algebra III, Proc. Conf. Kanpur(1968), Academia, Prague (1971), 301-306.

[22] R. Staum, The algebra of bounded continuous functions into a nonarchimedian field, Pacific J. Math., 50(1974), 169-185.

[23] L. A. Steen and J. A. Seebach Jr., Counterexamples in topology, Holt, Rinehart and Wiston, New York, 1970.

[24] M.Stone, Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41(1937), 374-481.

[25] A. Vadivel, V. Chandrasekar and M. Saraswathi, On contra α-$\mathcal{I}$-continuous functions, Int. J. Math. Anal., 47(4)(2010), 2329-2338.

[26] R. Vaidyanathaswamy, Set topology, Chelsea Publishing Company, New York, 1960.

[27] K. Viswanathan and J. Jayasudha, Another Decomposition of $A_{\mathcal{I}R}$-continuity, J. Adv. Stud. Top., 3(3)(2012), 1-5.

Received: September, 2012